**Homework Two**

Problem One:

Part A)

From a domain of 9 possible inputs there are a total of 39 outputs and when restricted to the commutative property our input space shrinks to a total of 6 different pairs of inputs that need to map to a unique output to form what will always be a surjective function.

The Input space for commutative outputs is the set {01 or 10, 0A or A0, A1 or 1A, 00, 11, AA}, each of these inputs has 3 possible outputs which are the set {0,1,A}. As a surjective function **There are 36 possible commutative function mappings.**

Part B)

If 0 were the identity element then we would have 1 possible mapping for each input combination with an identity element. There are 3 possible commutative input pairs left in the input space, where each of these input pairs have 3 possible outputs. This means when 0 is the identity element there are 1\*3\*3\*3 = 33 possible mappings.

For 1 or A being the identity elements the problem is the same so then

**There are a total of 3(33) = 34 possible function mappings with an identity element**

Part C) Assuming 0 and 1 are the identity elements for + and \* respectively…

|  |  |  |  |
| --- | --- | --- | --- |
| **+** | **0** | **A** | **1** |
| **0** | 0 | A | 1 |
| **A** | A | A | 1 |
| **1** | 1 | 1 | 1 |

|  |  |  |  |
| --- | --- | --- | --- |
| **\*** | **0** | **A** | **1** |
| **0** | 0 | 0 | 0 |
| **A** | 0 | A | A |
| **1** | 0 | A | 1 |

Part D)

**There are no possible 3 valued Boolean algebras in this system.**

For any 3 valued Boolean algebra, once we define two of our possible Boolean algebra values as one being the compliment of the other then we have a third Boolean algebra with no compliment. So for example if we choose 1 as the compliment of 0, then A has no compliments.

So then A+A’ and A\*A’ has no identity elements. If there are no identity elements for this operation then the whole boolean algebra system breaks down.

Problem Two:

Part A)

i.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Group | 0-Subcubes | 1-Subcubes | 2-Subcubes | 3-Subcubes |
| G0 | 0 | 0 | 0 | 0 |
| G1 | 3 | 0 | 0 | 0 |
| G2 | 0 | 0 | 0 | 0 |
| G3 | 1 | 0 | 0 | 0 |

Sum of Products: a’b’c + a’bc’ + ab’c’

Behavioral Description: Represents the functionality of the sum

ii.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Group | 0-Subcubes | 1-Subcubes | 2-Subcubes | 3-Subcubes |
| G0 | 0 | 0 | 0 | 0 |
| G1 | 0 | 0 | 0 | 0 |
| G2 | 3 | 3 | 0 | 0 |
| G3 | 1 | 0 | 0 | 0 |

Sum of Products: bc + ac + ab

Behavioral Description: Represents the functionality of the carry

iii.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Group | 0-Subcubes | 1-Subcubes | 2-Subcubes | 3-Subcubes |
| G0 | 0 | 0 | 0 | 0 |
| G1 | 2 | 2 | 0 | 0 |
| G2 | 1 | 1 | 0 | 0 |
| G3 | 1 | 0 | 0 | 0 |

Sum of Products: ac’ + bc’ + ab

Behavioral Description: Represents the functionality of a borrow out

Part B)

The sum function can be represented by an XOR of all of the respective inputs. Namely,

Sum = Cin ^ Din ^ V ^ W ^ X ^ Y ^ Z where “^” denotes the XOR operator.

This function is only true for an odd number of 1’s in the string, implying that groups G1, G3, G5 and G7 are the only groups which contain prime implicants in Subcube 0 because each group has no adjacent groups.

The total number of prime implicants are as follows

G1 -> 7, G3 -> , G5->, G7-> all in subcube 0

**So there are a total of 7+++1 prime implicants.**

Examples of prime implicants are as follows (all in subcube 0):

For G1: 0100000, For G3: 0110100, For G5: 0111101, For G7: 1111111

Part C)

A double carry is only generated when there are 4 or more ones in the input space. This means that the double carry function is defined only in groups G4, G5, G6, G7 in subcube 0. Due to the nature of the tabulation method, the implicants reduce to residing in subcube 3, Group 4. Groups 5, 6 and 7 drop out as we simplify to larger and large cubes. The table for DC is as follows:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Group | Subcube 0 | Subcube1 | Subcube2 | Subcube3 | Subcube4 | Subcube5 |
| G0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G1 | 0 | 0 | 0 | 0 | 0 | 0 |
| G2 | 0 | 0 | 0 | 0 | 0 | 0 |
| G3 | 0 | 0 | 0 | 0 | 0 | 0 |
| G4 |  |  |  |  | 0 |  |
| G5 |  |  |  | 0 | 0 | 0 |
| G6 |  |  | 0 | 0 | 0 | 0 |
| G7 |  | 0 | 0 | 0 | 0 | 0 |

The value in any cell is the multinomial represented in the following way:

**So The total number of prime implicants in this table are equal to the amount contained in group 4, subcube 3 which is**

An example of a prime implicant is as follows:

Group 4, Subcube 3: 11- -1-1

Part D)

A carry only happens when there are two ones being added. The carry bit is on at the end of the addition for the (DC)(C)(S) bit patterns corresponding to 010,011,110,111. This means that when there are two ones, three ones, six ones or seven ones a carry event will occur.

Knowing this or tabulation comes out to be the following: (on the next page)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Group | Subcube 0 | Subcube1 | Subcube2 | Subcube3 | Subcube4 | Subcube5 |
| G0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G1 | 0 | 0 | 0 | 0 | 0 | 0 |
| G2 |  |  | 0 | 0 | 0 | 0 |
| G3 |  | 0 | 0 | 0 | 0 | 0 |
| G4 | 0 | 0 | 0 |  | 0 |  |
| G5 | 0 | 0 | 0 | 0 | 0 | 0 |
| G6 |  |  | 0 | 0 | 0 | 0 |
| G7 |  | 0 | 0 | 0 | 0 | 0 |

This tabulation shows that we have prime implicants in Subcube 1, Group 2 and Subcube 1, Group 6.

The total amount of prime implicants is

An example of a prime implicant in Subcube 1, Group 2 is 010010-

and an example of one in Subcube 1, Group 6 is 111-111

Problem Three:

Part A)

Other adjacent minterms that fail to lie next to each other are xxxdef’ and xxxd’ef’

Part B)

Here is one of the possible tables with all hamming adjacent minterms:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| abc/def | 101 | 111 | 110 | 010 | 000 | 001 |
| 101 |  |  |  |  |  |  |
| 111 |  |  |  |  |  |  |
| 110 |  |  |  |  |  |  |
| 010 |  |  |  |  |  |  |
| 000 |  |  |  |  |  |  |
| 001 |  |  |  |  |  |  |

The way this was constructed was by analyzing the set of all possible 3 variable combinations (8 total) 000,001,010,011,100,101,110,111. When looking at this it is possible to notice that way may construct a table by eliminating any two of these combinations that are hamming distance 3 apart. Any two minterms that have a hamming distance of three in either the row variables or column variables would mean that it would be impossible to show them in the map in the first place. The problem in Part A will not manifest itself since in this new configuration all of the minterms are now hamming adjacent. The minterms excluded in part A were only hamming distance one apart as well, and excluding those would not provide every possible minterm for our map.

Part C)

In this part, it is the third student that is right. The only minterms that can be circled are the ones spanning two columns and two rows. The cases where they are spanning a row or a line encounters a ‘wrap-around’ problem.

For a prime implicant of size 4 to exist, every implicant must be hamming adjacent to two other implicants and hamming distance 2 away from a third implicant. In the case of the prime implicant spanning two columns and rows, these conditions are satisfied. In the case of the 4 implicants spanning a line, we find the the implicants on the ends are not adjacent to at least two other implicants and therefore are not valid prime implicants. The line of implicants would have worked if an implicant at an edge could wrap around to the implicant at the other edge.

Problem 4:

Part A)

i. This table has no valid karnaugh maps, row P0 only has 3 minterms. For an implicant to be prime, the number of minterms must be a power of 2.

ii. There are no valid maps for this table. Row P1 is dominated by Row P0, this indicated that Row P1 is not a prime implicant. Since a Quine-McClusky table should only contain prime implicants after tabulation, we can see that this is an invalid table.

iii. There are no valid maps for this table. The set of minterms in this table is circular and when we map them, we find that they all represent one large prime implicant when combined. ML,,Mk, Mi , Mj  each have two implicants in the table which are adjacent and on which is only hamming distance 2 away which means together they form one prime implicant.

Part B)

i. This is possible with the following 4 variable K-Map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 00 | 01 | 11 | 10 |
| 00 | Mn | X | Mk |  |
| 01 |  | Mi | Mj |  |
| 11 | Mo | Ml | X | Mp |
| 10 |  |  |  |  |

ii. This is possible with the following 4-variable K-Map

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 00 | 01 | 11 | 10 |
| 00 | Mi | Mj | X | X |
| 01 |  |  | Mn | Mo |
| 11 | Mk | Ml |  |  |
| 10 | X | X |  |  |

iii. This one is possible with the following 5-variable K-Map.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| | | | X | | X | | | X | |
| | X | X | Mi | X | Mj | Ml | X | Mn | |
| | X | | X | | Mk | | X | Mo | |
| | | | X | | X | | | X | |
| | | | | | | | | | |

(\*Note that the ‘|’ character divides the top layer from the bottom layer)